Introduction to optimisation

Recitation 01
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Question 1 *Lipton produces two types of beverages. Let us call them A and B. To do this, they purchase intermediate stage products in bulk and transform those into the final drinks A and B before distributing those drinks through various channels. The main constraint of Lipton is production capacity. Product A requires* 3 *machine hours per liter, but because of additional requirements, product B requires* 4 *hours of machine time per liter. The Lipton factory is limited to a total of 20,000 machine hours. The direct operating cost of product A is* \$5 *per liter while the operating cost for product B is* \$3 *per liter. Lipton's funds that are available to finance direct costs are set to \$40,000 in total. On top of this, the management expects that* 40% *of the sales of product A and* 35% *of the sales of product B can be collected during the production period and used to finance the operations. Product A is sold to the distributors for* \$6 *per liter while product B is sold to the distributors for* \$4.50 *per liter. The Lipton factory is currently closed because the production and marketing team on the one hand and the finance team on the other disagree on the spending of an additional* \$250 *for machine repairement. It has been estimated that such an additional expense could lead to an increase in capacity of 2000 machine hours but that the machines would have to be repaired again at the end of the period.*

- *Formulate this problem as a linear program in the two variables x_A and x_B, keeping in mind that Lipton's objective is to maximize the profit which corresponds to revenues minus the costs.*
- *Plot the set of feasible solutions (solutions x_A, x_B that satisfy the inequalities of your LP)*
- *Plot the isoprofit line $x_A + 1.5x_B = \$6000$. Is there any feasible production (x_A, x_B) that leads to a profit of* \$6000*? How could you find a production (x_A, x_B) that would maximize the profit?*

Question 2 *Turn the following LP into standard form*

$$\begin{aligned} \max \quad & 4x_1 - 2x_3 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 = 2 \\ & x_1 - 2x_2 - x_3 \leq 1 \\ & x_1 + x_3 \geq -2 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

Question 3 *Find the solution of the following linear program using the graphical method*

$$\begin{aligned} \max \quad & 3x + 2y \\ \text{s.t} \quad & x + 2y \leq 4 \\ & x - y \leq 1 \\ & x \geq 0, y \geq 0 \end{aligned}$$

Question 4 *Find the minimum value of* $5x + 7$ *where* $x \geq 0, y \geq 0$ *subject to the constraints*

$$\begin{aligned} & 2x + 3y \geq 3 \\ & 3x - y \leq 15 \\ & -x + y \leq 4 \\ & 2x + 5y \leq 27 \end{aligned}$$

Question 5 *Write the following program in standard form and solve it using the graphical method*

$$\begin{aligned} \max \quad & x_1 + 2x_2 \\ \text{s.t} \quad & x_2 \leq 2x_1 + 2 \\ & x_1 + 3x_2 \leq 27 \\ & x_1 + x_2 \leq 15 \\ & 2x_1 \leq x_2 + 18 \\ & x \geq 0 \end{aligned}$$

Question 6 *Write the following linear programs in standard form*

$$\begin{aligned} \max \quad & 2x_1 + 3x_2 + 4x_3 + 5x_4 \\ \text{s.t} \quad & x_1 + x_2 - x_3 + x_4 \leq 10 \\ & x_1 + 2x_2 \leq 8 \\ & x_3 + x_4 \leq 20 \\ & x_i \geq 0 \end{aligned}$$

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\begin{aligned}
\min \quad & 3x_1 - x_2 \\
\text{s.t} \quad & -x_1 + 6x_2 - x_3 + x_4 \geq -3 \\
& 7x_2 + x_4 = 5 \\
& x_3 + x_4 \leq 2 \\
& -1 \leq x_2, x_3 \leq 5, -2 \leq x_4 \leq 2
\end{aligned}
$$

Question 7 (Exercise 1.9 in [1]) *Consider a school district with I neighborhoods, J schools and g grades at each school. Each school j has a capacity of C_{jg} students for grade g. In each neighborhood i, the student population of grade i is S_{ig}. Finally the distance of school j from neighborhood i is d_{ij}. Formulate a linear program whose objective is to assign all the students to schools while minimizing the total distance traveled by all the students.*

Question 8 *Find the solution of the following LP (using the Simplex algorithm)*

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\begin{aligned}
\min \quad & 3x_1 + x_2 + 9x_3 + x_4 \\
\text{s.t} \quad & x_1 + 2x_3 + x_4 = 4 \\
& x_2 + x_3 - x_4 = 2 \\
& x_i \geq 0.
\end{aligned}
$$

Question 9 (Exercise 1.14 in [1]) *A company produces and sells two different products. The demand for each product is unlimited, but the company is constrained by cash availability and machine capacity. Each unit of the first and second product requires 3 and 4 machine hours, respectively. There are 20,000 machine hours available in the current production period. The production costs are \$3 and \$2 per unit of the first and second product, respectively. The selling prices of the first and second product are \$6 and \$5.40 per unit, respectively. The available cash is \$4,000; furthermore, 45% of the sales revenues from the first product and 30% of the sales revenues from the second product will be made available to finance operations during the current period.*

(a) Formulate a linear programming problem that aims at maximizing net income subject to the cash availability and machine capacity limitations.

(b) Solve the problem graphically to obtain an optimal solution.

(c) Suppose that the company could increase its available machine hours by 2,000, after spending \$400 for certain repairs. Should the investment be made?

References

[1] Dimitris Bertsimas, and John N Tsitsiklis, *Introduction to linear optimization.* Vol. 6. Athena scientific Belmont, MA, 1997.